Introduction to Econometric Analysis

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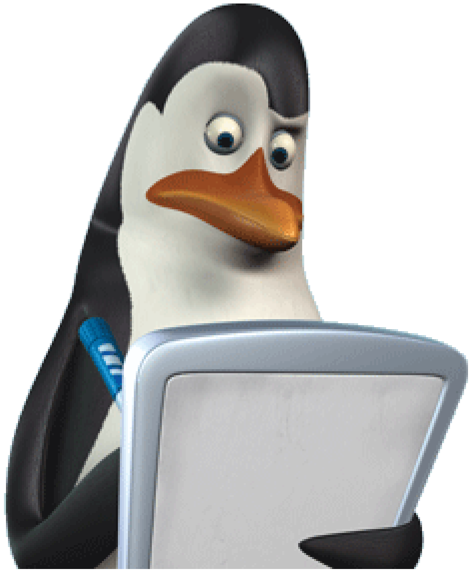
References

- Greene W. H., Econometric Analysis, 7th ed., Prentice Hall, 2011.
- Murray M. P. (2005) Econometrics: A Modern Introduction. Prentice Hall.
- Stock James H., Mark W. Watson (2010) Introduction to Econometrics (3rd Edition)

Agenda

- Econometric review
- Econometric tests

ECONOMETRIC REVIEW



Econometric analysis

- Theoretical approach
- Empirical approach

Types of Data and Notation

- ✓ Time series data
- ✓ Cross-sectional data
- ✓ Panel data, a combination of mentioned above types

Time series data

- **The data may be**
  - quantitative (e.g. exchange rates, stock prices, number of shares outstanding),
  - qualitative (e.g. day of the week).

- **Examples of time series data**

Series	**Frequency**
GNP or unemployment	monthly or quarterly
government budget deficit	annually
money supply	weekly
value of a stock market index	as transactions occur

Examples of Problems Using Time Series Regression

1. How the value of a country's stock index has varied with that country's macroeconomic fundamentals.
2. How the value of a company's stock price has varied when it announced the value of its dividend payment.
3. The effect on country's currency of an increase in its interest rate.

Cross-sectional data

- Cross-sectional data is data on one or more variables collected at a single point in time, e.g.
  - A poll of usage of internet stock broking services
  - Cross-section of stock returns on the New York Stock Exchange
  - A sample of bond credit ratings for UK banks

Examples of Problems Using a Cross-Sectional Regression

- The relationship between company **size** and the **return** to investing in its shares
- The relationship between a country's **GDP level** and the **probability** that the government will **default** on its sovereign debt.

Panel Data

- Panel Data has the dimensions of both time series and cross-sections, e.g. the ***daily prices of number of blue chip stocks over two years***.

- It is common to denote that each observation by the letter **t** and the total number of observations by **T** for time series data, and to denote each observation by the letter **i** and the total number of observations by **N** for cross-sectional data.

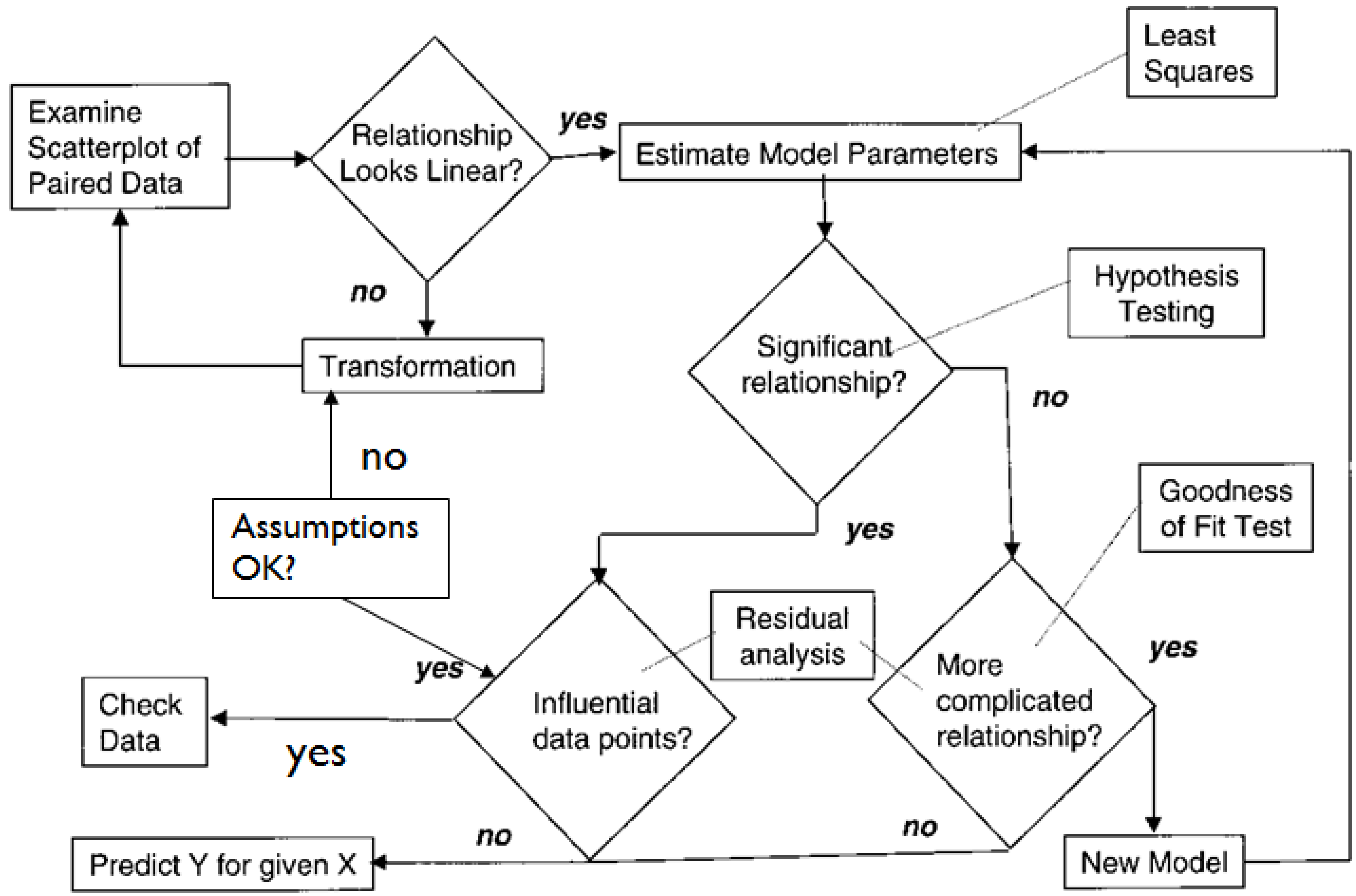
Goal

- Develop **a statistical model** that can predict the values of a dependent (response) variable based upon the values of the independent (explanatory) variables.

Regression Modeling Steps

- Define a problem or question
- Specify model
- Collect data
- Do descriptive data analysis
- Estimate unknown parameters
- Evaluate model
- Use model for prediction

How is a Linear Regression Analysis done?



Linear regression

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{1t} + \ldots + \beta_{k-1} x_{k-1t} + \varepsilon_t, t = \overline{1, n}$$

y_t - dependent variable;

$x_{1t}, x_{2t}, \ldots, x_{k-1t}$ - independent variables;

ε_t - residuals.

Assumptions

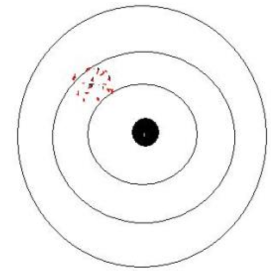
- **Linearity** - the Y variable is linearly related to the value of the X variable.
- **Independence of Error** - the error (residual) is independent for each value of X.
- **Homoscedasticity** - the variation around the line of regression be constant for all values of X.
- **Normality** - the values of Y be normally distributed at each value of X.

Method of Least Squares

- The **straight line** that best fits the data.
- Determine the straight line for which the differences between the actual values (Y) and the values that would be predicted from the fitted line of regression (Y-hat) are as small as possible.

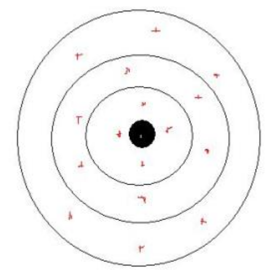
$$L = \sum_{i=1}^{n} \varepsilon_i^{\ 2} = \sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{j=1}^{k} \beta_j x_{ij} \right)^2 \to \min$$

The Three Desirable Characteristics

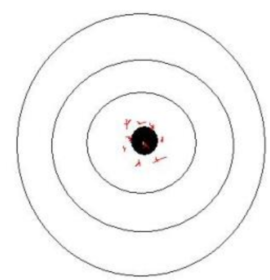


- **Lack of bias** $E(\hat{\beta}) = \beta$
- **Efficiency**
  - Standard error will be minimum
    - Remember:

$$\operatorname{var}(\hat{\beta}) = \frac{1}{\sum x_i^2}\sigma^2 = \frac{\sigma^2}{\sum x_i^2}$$



    - OLS will minimize σ^2 (the error variance)



- **Consistency**
  - As N increases the standard error decreases
    - *Notice: as N increases so does Σx_i^2*

Inherently Linear Models

- Non-linear models that can be expressed in linear form
  - *Can be estimated by least square in linear form*
- Require data transformation

Dummy-Variable Regression Model

- Involves categorical X variable with two levels
  - *e.g., female-male, employed-not employed, etc.*
- Variable levels coded 0 & 1
- Assumes only intercept is different
  - *Slopes are constant across categories*

ECONOMETRIC TESTS

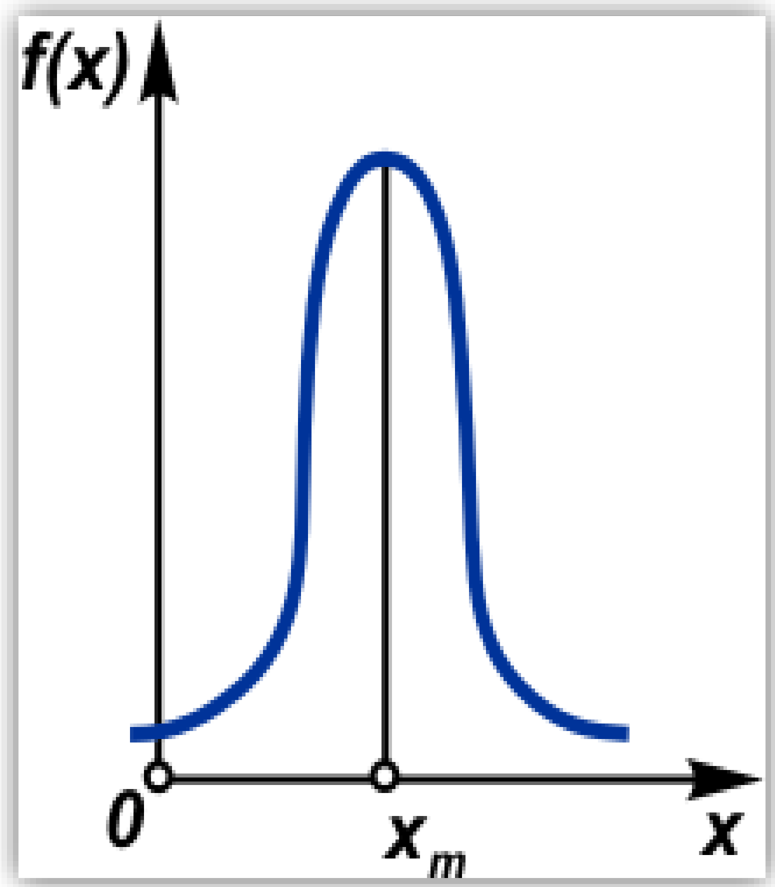
Multiple Regression Tests

- Test residual for normality
- Test parameter significance
  - Overall model
  - Individual coefficients
- Test for multicollinearity
- Test for model stability
- Test for residuals autocorrelation
- Test for residuals homoscedasticity
- Test for specification
- Test for stationary process

Test residual for normality

Check normality of residuals:

- Jarque-Bera statistics
- Shapiro–Wilk test



Jarque-Bera statistics

$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4}\left((K-3)^2\right)\right)$$

$$S = \frac{\hat{\mu}_3}{\hat{\sigma}^3} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^{3/2}}$$

$$K = \frac{\hat{\mu}_4}{\hat{\sigma}^4} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^{2}}$$

- ***S*** *is the sample skewness,*
- ***K*** *is the sample kurtosis.*

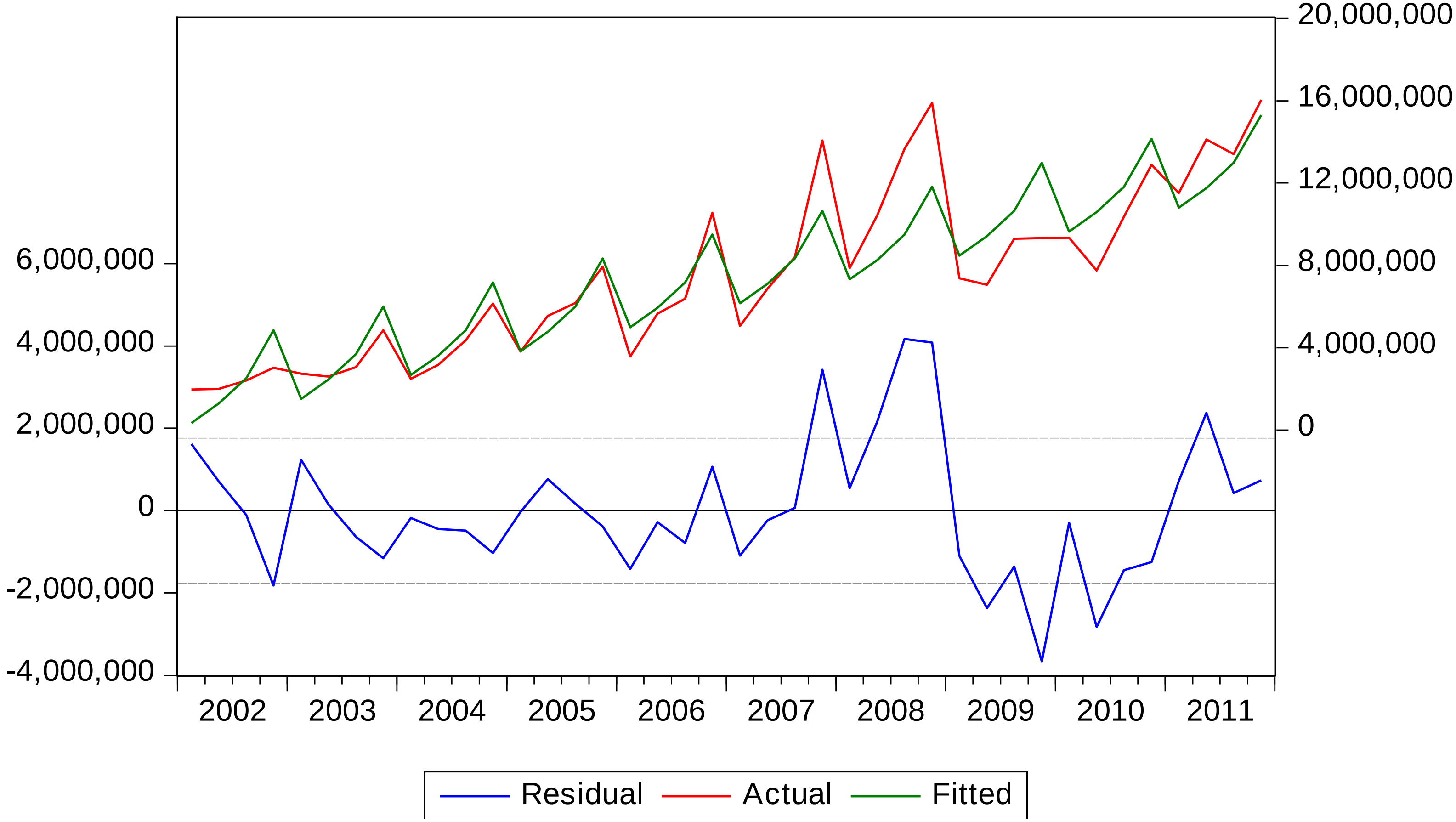
Example

Dependent Variable: TAX_ENT
Method: Least Squares
Date: 12/09/12 Time: 20:49
Sample: 2002Q1 2011Q4
Included observations: 40

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	3973770.	754540.7	5.266475	0.0000
@TREND	290525.1	24239.34	11.98568	0.0000
@SEAS(1)	-3627516.	791034.8	-4.585786	0.0001
@SEAS(2)	-2975920.	789175.7	-3.770922	0.0006
@SEAS(3)	-2032456.	788058.1	-2.579068	0.0143

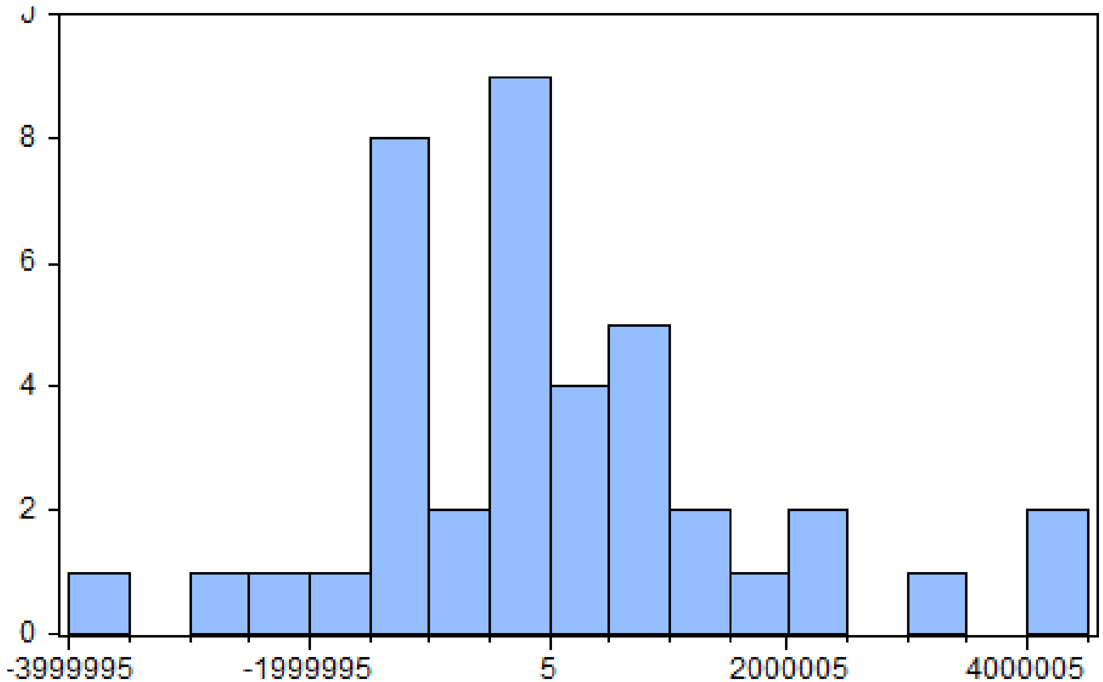
R-squared	0.837415	Mean dependent var	7480035.
Adjusted R-squared	0.818834	S.D. dependent var	4138083.
S.E. of regression	1761318.	Akaike info criterion	31.71749
Sum squared resid	1.09E+14	Schwarz criterion	31.92860
Log likelihood	-629.3498	Hannan-Quinn criter.	31.79382
F-statistic	45.06800	Durbin-Watson stat	1.123746
Prob(F-statistic)	0.000000		

Residuals



20,000,000
16,000,000
12,000,000
8,000,000
4,000,000
0
6,000,000
4,000,000
2,000,000
0
-2,000,000
-4,000,000
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
Residual
Actual
Fitted

Check for normality



Series: Residuals
Sample 2002Q1 2011Q4
Observations 40

Mean	-3.49e-11
Median	-204126.5
Maximum	4171075.
Minimum	-3659494.
Std. Dev.	1668551.
Skewness	0.590368
Kurtosis	3.696860
Jarque-Bera	3.132917
Probability	0.208783

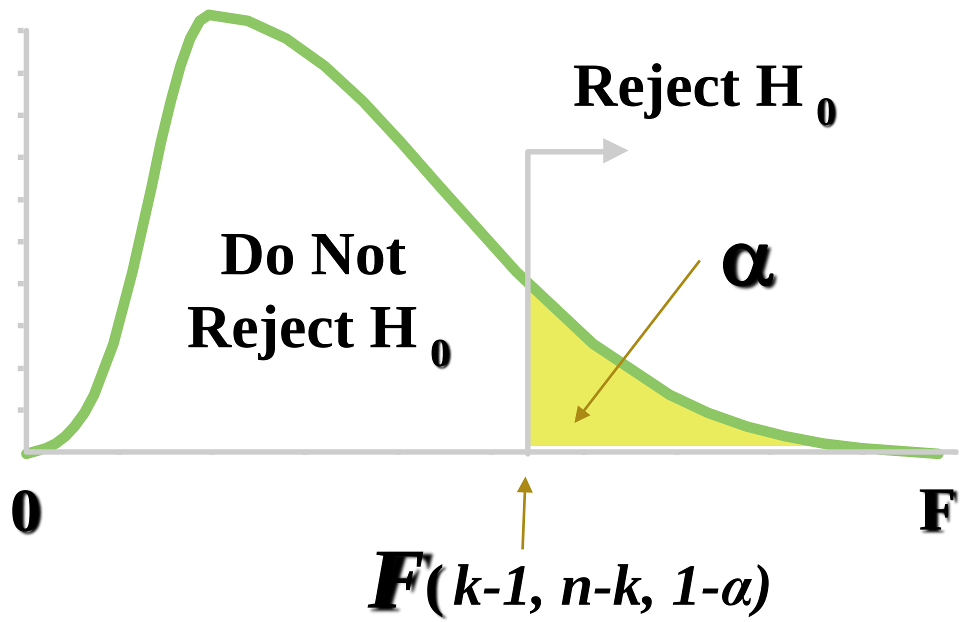
Test parameter significance: Overall model

- Hypotheses
  - H_0: $\beta_1 = \beta_2 = \ldots = \beta_{k-1} = 0$
    - *No Linear Relationship*
  - H_a: At Least One Coefficient Is Not 0
    - *At Least One X Variable linearly Affects Y*

$$F = \frac{RSS/(k-1)}{ESS/(n-k)} = \frac{R^2/\left(k-1\right)}{\left(1-R^2\right)/\left(n-k\right)} \overset{H_0}{\sim} F_{k-1,n-k}$$

Overall Significance Rejection Rule

- Reject H_0 in favor of H_a if F_{calc} falls in colored area



- Reject H_0 for H_a if P-value = $P(F>F_{calc})<\alpha$

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Test of slope coefficients

- Hypotheses
  - H_0: $\beta_i = m$
  - H_a: $\beta_i \neq m$

Slope Coefficient Test Statistic

$$t = \frac{\hat{\beta}_i - m}{S_{\hat{\beta}_i}}$$

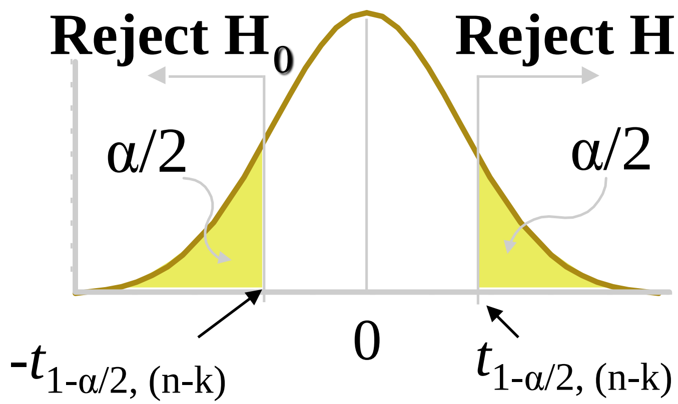
where $$S_{\hat{\beta}_i} = \frac{S}{\sqrt{\sum_{i=1}^{n} X_i^2 - \frac{\left(\sum_{i=1}^{n} X_i\right)^2}{n}}}$$

with $$S = \hat{\sigma} = \sqrt{\frac{RSS}{n-k}}$$

and $$RSS = \sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2 = \sum_{i=1}^{n}\left[Y_i - \left(\hat{\beta}_0 + \sum_{i=1}^{k-1}\hat{\beta}_i X_i\right)\right]^2$$

Test of Slope Coefficient Rejection Rule

- Reject H_0 in favor of H_a if t falls in colored area

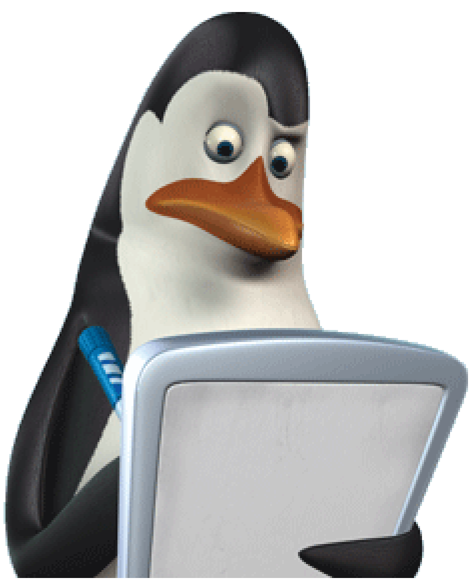


$T = t_{(n\text{-}k)}$

- Reject H_0 for H_a if P-value = $P(T>|t|) < \alpha$

Special case: significance of coefficient

- Hypotheses
  - H_0: $\beta_i=0$
  - H_a: $\beta_i \neq 0$



$$\boxed{t = \frac{\hat{\beta}_i}{S_{\hat{\beta}_i}}}$$

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Wald test

Null Hypothesis: $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$

Alternative hypothesis $H_1 : \beta_1$ or β_2 or β_3

or any two of them or all are nonzero.

At least one of them is significant.

In matrix notation

Hypothesis: $Rb = r \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Test statistics with J numbers of restriction

$$F = \frac{\dfrac{(Rb-r)'\left[R\operatorname{cov}(b)R'\right]^{-1}(Rb-r)}{J}}{\dfrac{RSS}{n-k}}$$

Calculate F and compare it with the critical values F(J, n-k) from the Table.

Test for multicollinearity

- High **correlation** between X variables
- Coefficients measure **combined effect**
- Leads to **unstable** coefficients depending on X variables in model
- **Always** exists; matter of degree
- *Example*: Using both total number of rooms and number of bedrooms as explanatory variables in same model

Detecting Multicollinearity

- Farrar-Glauber Multicollinearity
- VIF-test

- Few remedies
  - Obtain new sample data
  - Eliminate one correlated X variable
  - Standardize your independent variables.

Example

$$\hat{s}_t = 0.4 + 0.8y_t + 0.2li_t - 0.1si_t$$
$$(0.9) \quad (1.2) \quad (0.4) \quad (0.1)$$

$\overline{\mathrm{R}}^2 = 0.98$, (standard errors in parentheses)

(n = 60). where:

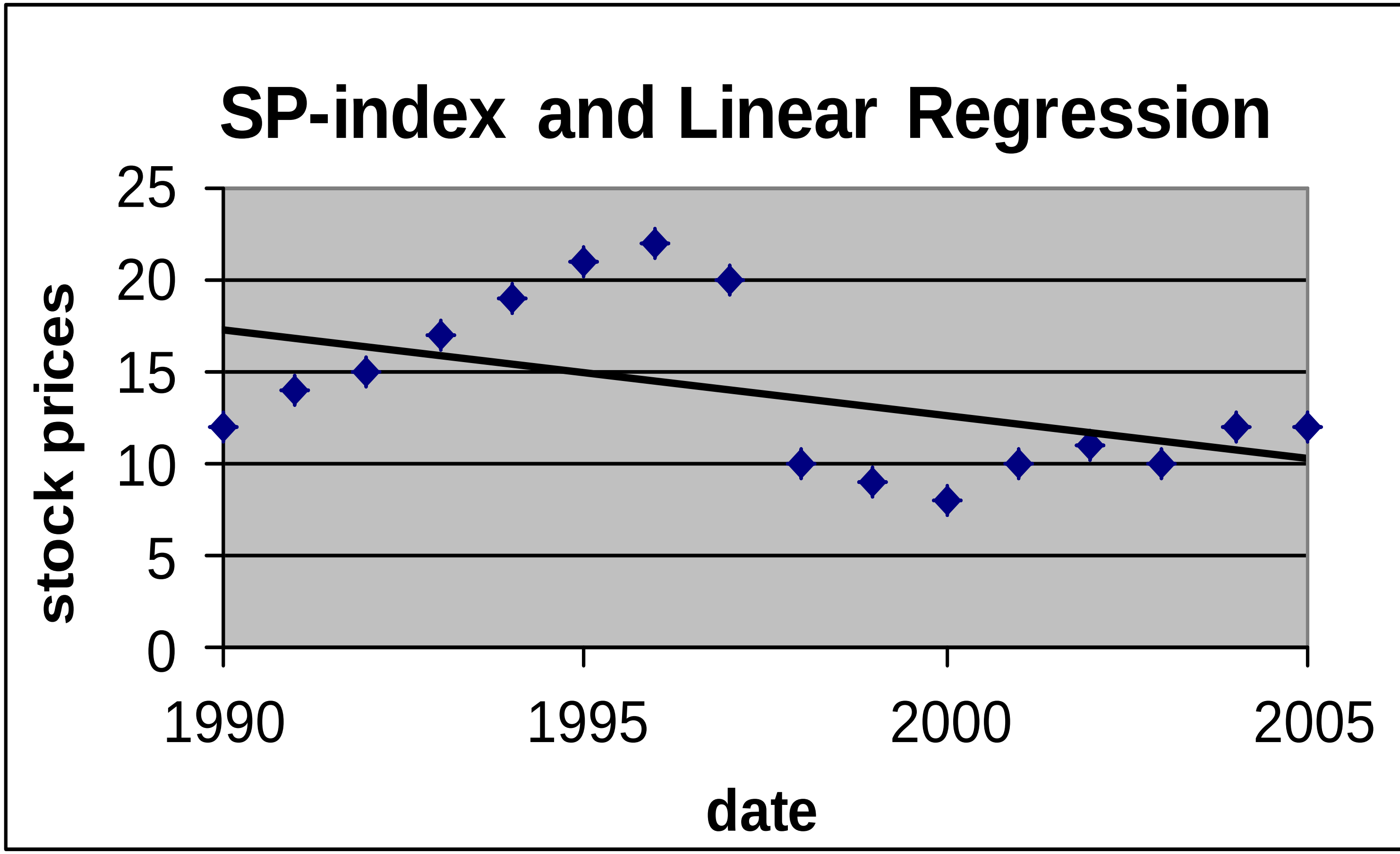
s_t – stock prices

y_t – output

li_t – long-run interest rates

si_t - short-run interest rates

Test for structural breaks



Chow Test

- Tests whether the coefficients in two linear regressions on different data sets are equal.

$$F = \frac{RSS_c - (RSS_1 + RSS_2)/k}{\left(RSS_1 + RSS_2\right)/n - 2k} \sim F_{k,n-2k}$$

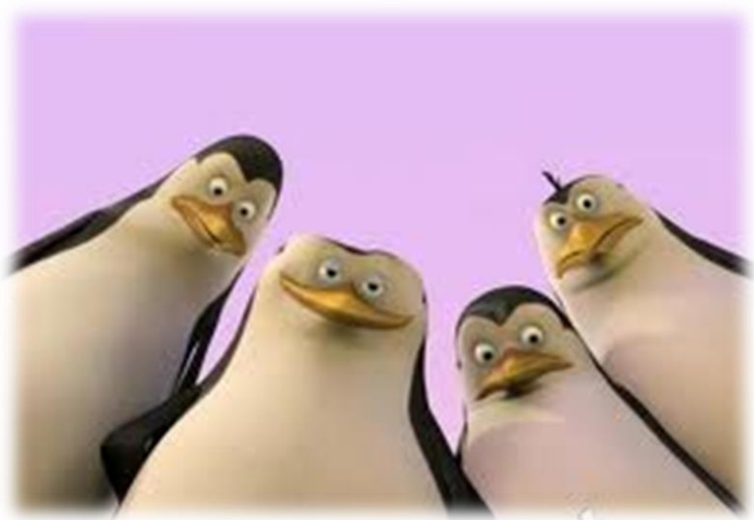
$$RSS_c - combined\_RSS$$

$$RSS_1 - pre - break\_RSS$$

$$RSS_2 - post - break\_RSS$$

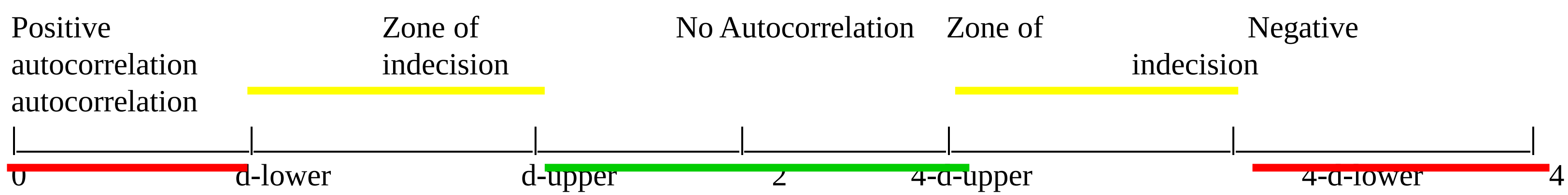
Test for residuals autocorrelation

- **Durbin-Watson test** (only checks for first order serial correlation in residuals)
- **Breusch-Godfrey Test** (checks for higher order autocorrelation AR(q) in residuals)



Durbin-Watson statistic

$$d = \frac{\sum (e_i - e_{i-1})^2}{\sum e_i^2}, \text{ for n and K - 1 d.f.}$$



- Autocorrelation is clearly evident
- Ambiguous – cannot rule out autocorrelation
- Autocorrelation in not evident

Breusch-Godfrey Test

Higher Order Autocorrelation model : AR(p)

$$\mu_t = \rho_1\mu_{t-1} + \rho_2\mu_{t-2} + \ldots + \rho_p\mu_{t-p} + \varepsilon_t$$

Null Hypothesis

$$H_0 : \rho_1 = \rho_2 = \ldots = \rho_p = 0$$

Test Model:

$$\hat{\mu}_t = \delta_1 + \delta_2 X_{2t} + \ldots + \delta_k X_{kt} + \lambda_1\hat{\mu}_{t-1} + \ldots + \lambda_p\hat{\mu}_{t-p} + \omega_t$$

Test Statistic

$$LM = (n - p) * R^2_{aux} \sim \chi^2_p$$

Tests for Heteroskedasticity

- There are two types of tests:
  - Tests for continuous changes in variance: ***White test, Breusch–Pagan tests, etc***.
  - Tests for discrete (lumpy) changes in variance: ***the Goldfeld–Quandt test***

The White Test

- The White test for heteroskedasticity has a basic premise: ***if disturbances are homoskedastic, then squared errors are on average roughly constant.***
- Explanators should **NOT** be able to predict squared errors, or their proxy, squared residuals.
- The White test is the most general test for heteroskedasticity.

Steps of the White Test

- Regress Y against your various explanators using OLS, compute the OLS residuals, $\varepsilon_1, \ldots, \varepsilon_n$
- Regress ε_i^2 against a constant, all of the explanators, the squares of the explanators, and all possible interactions between the explanators (p slopes total)
- Compute R^2 from the "auxiliary equation" in step 2
- Compare nR^2 to the critical value from the Chi-squared distribution with p degrees of freedom.

The Breusch–Pagan Test – 1

- The Breusch–Pagan test is very similar to the White test.
- The White test specifies exactly which explanators to include in the auxiliary equation. Because the test includes cross-terms, the number of slopes (p) increases very quickly.
- In the Breusch–Pagan test the econometrician selects which explanators to include. Otherwise, the tests are the same.

The Breusch–Pagan Test – 2

- In the Breusch–Pagan test, the econometrician selects **m** explanators to include in the auxiliary equation.
- Which explanators to include is a **judgment call**.
- A **good** judgment call leads to a more powerful test than the White test.
- A **poor** judgment call leads to a poor test.

The Goldfeld–Quandt Test – 1

- Both the ***White*** test and ***the Breusch–Pagan*** test focus on smoothly changing variances for the disturbances.
- ***The Goldfeld–Quandt*** test compares the variance of error terms across discrete subgroups.
- Under homoskedasticity, all subgroups should have the same estimated variances.

The Goldfeld–Quandt Test – 2

Divide the n observations into h groups, of sizes $n_1..n_h$

Choose two groups, say 1 and 2.

$$H_0 : \sigma_1^{\ 2} = \sigma_2^{\ 2} \quad \text{against } H_a : \sigma_1^{\ 2} \neq \sigma_2^{\ 2}$$

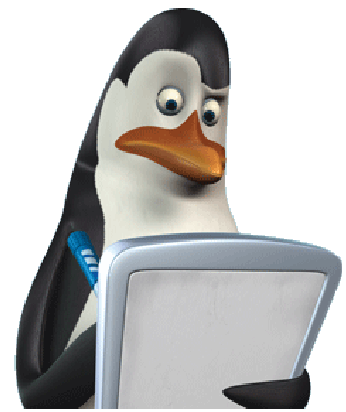
Regress Y against the explanators for group 1.

Regress Y against the explanators for group 2.

Goldfeld–Quandt Test – 3

Relabel the groups as L and S, such that $\dfrac{RSS_L}{n_L - k} > \dfrac{RSS_S}{n_S - k}$

Compute $F_{calc} = \dfrac{\dfrac{RSS_L}{n_L - k}}{\dfrac{RSS_S}{n_S - k}} > 1$



Compare F_{calc} to the critical value for an F-statistic with $(n_L - k)$ and $(n_S - k)$ degrees of freedom.

Test for specification

$$F^{k}_{n-m-k+1} \sim \boxed{\dfrac{\dfrac{R_1^2 - R_0^2}{k}}{\dfrac{1-R_1^2}{n-m-k}}}$$

Ramsey's RESET

- RESET relies on a trick similar to the special form of the White test
- Instead of adding functions of the x's directly, we add and test functions of ŷ
- So, estimate $y = \beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k + \delta_1 \hat{y}^2 + \delta_2 \hat{y}^3 + \varepsilon$ and test

H_0: $\delta_1 = 0$, $\delta_2 = 0$ using $F \sim F_{2,n-k-3}$ or $LM \sim \chi^2(2)$.

Stationary process

- A **stationary process** is a stochastic process whose joint probability distribution does not change when shifted in time.
- Parameters such as the mean and variance, if they are present, also do not change over time and do not follow any trends.

Solutions:

- Taking differences (Dickey-Fuller test)
- Trend-stationary processes

"What should we do, if we fail to find an appropriate model that satisfy all tests?"

Question

REVIEW

Linear regression

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{1t} + \ldots + \beta_{k-1} x_{k-1t} + \varepsilon_t, t = \overline{1,n}$$

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ε_t - residuals.

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- Do descriptive data analysis
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QUESTIONS?

THANK YOU FOR YOUR ATTENTION!